Scattering and Modulational Instabilities in Magnetized Plasmas

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The decay of a high-frequency wave into a scattered and an electrostatic wave is investigated for a homogeneous magnetized plasma. For wave propagation in arbitrary directions, an equation for the scattered wave is obtained accounting for the effect of the non-linear current density produced by the three-wave interaction process. As an illustration, the propagation of electromagnetic waves perpendicular to the external magnetic field is considered. The growth rates and thresholds for the stimulated scattering and modulational instabilities are obtained. The influence of a weak inhomogeneity is also considered.

1. Introduction

The problem of parametric decay of an intense coherent electromagnetic (EM) wave into another EM and an electrostatic (ES) wave has recently received much attention in both laser fusion and ionospheric heating experiments [1]. For unmagnetized plasmas, it has been shown theoretically [2] and by computer simulation [3] that the secondary EM wave can backscatter from the plasma and hence carry away a considerable fraction of the pump wave energy. The scattering instabilities associated with this process can then prevent the EM radiation from being anomalously absorbed by the plasma.

Recently, Shukla et al. and Lee have investigated in detail the decay of a circularly polarized wave into an EM wave and an ES wave for waves propagating *parallel* to an external magnetic field [4]. Lee obtained the growth rate and the threshold for the stimulated Raman scattering instability by means of a multiple time-scale expansion method, while Shukla et al. used the much simpler method of Drake et al. [2] to discuss the stimulated Brillouin scattering instability.

In this paper, we consider the decay of a high-frequency wave into a scattered wave and an electrostatic wave for the general case of waves propagating in *arbitrary* directions with respect to the external magnetic field $B_0 \hat{z}$. We develop an equation for the scattered wave, including the effect of the non-linear current density originating from the interaction between the three waves. This equation, from which the dispersion relation for the excited waves can readily be obtained, is the basis for studying parametric scattering and modulational instabilities in magnetized plasmas. As an example, we investigate the particularly simple yet practical problem [5] in which the pump and the scattered wave are ordinary EM waves propagating *perpendicular* to the magnetic field. Several frequency ranges of the ES wave are considered. The growth rates and thresholds for the corresponding scattering and modulational instabilities are significantly modified by the external magnetic field. When a weak density gradient is present, we show that the pump can also scatter off drift waves.

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2. Basic Equations

Consider a high-frequency pump wave

$$\boldsymbol{E}_0 = \boldsymbol{E}_{0+} + \boldsymbol{E}_{0-} \tag{1}$$

propagating in an arbitrary direction $\hat{k}_0$ with respect to the external magnetic field, such that $E_{0\pm} \propto \exp[\mp i(\boldsymbol{k}_0 \cdot \boldsymbol{r} - \omega_0 t)]$. The pump frequency ω_0 and the wave vektor $\boldsymbol{k}_0$ satisfy the usual linear magnetized plasma dispersion relation [6], which also determines the pump polarization and hence the exact form of $\boldsymbol{E}_{0\pm}$. Thus, except for some special directions of propagation, the pump is a mixed EM-ES mode.

We shall use the fluid equations for the electrons and ions, as well as Maxwell's equations, namely,

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \boldsymbol{v}_j) = 0 , \tag{2}$$

$$\frac{\partial \boldsymbol{v}_j}{\partial t} + \boldsymbol{v}_j \cdot \nabla \boldsymbol{v}_j = \frac{q_j}{m_j} (\boldsymbol{E} + \boldsymbol{v}_j \times \boldsymbol{B}/c) - \gamma_j v_{Tj}^2 \nabla \ln n_j , \tag{3}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c} \frac{\partial \boldsymbol{B}}{\partial t} , \tag{4}$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi}{c}\sum_{j=\mathrm{e,i}} q_j n_j \boldsymbol{v}_j, \tag{5}$$

$$\nabla \cdot \boldsymbol{E} = 4\pi \sum_{j=\mathrm{e,i}} q_j n_j, \tag{6}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{7}$$

where q_j and m_j are the charge and mass of species $j=\mathrm{i,e}$ $(q_\mathrm{e}=-e)$; c is the velocity of light, $v_{\mathrm{T}j}=(T_j/m_j)^{1/2}$ is the thermal velocity, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields.

The pump wave excites an ES mode $(\omega_l, \boldsymbol{k}_l)$ and sideband modes $(\omega_\pm=\omega_l\pm\omega_0,\ \boldsymbol{k}_\pm=\boldsymbol{k}_l\pm\boldsymbol{k}_0)$. The latter are also in general mixed EM-ES waves and they interact with the pump wave to produce a low frequency ponderomotive force which can enhance the original density oscillations of the electrostatic wave. On the other hand, the interaction between the pump and the ES wave produces a force which enhances the sideband modes. We shall investigate the dispersion of the scattered-off wave. Only the lowest order coupling between the pump and the excited waves is considered. We assume that only the electrons respond to the high-frequency fields, whereas the ions also participate in the low-frequency oscillations.

Combining Eqs. (4), (5), and (7), one obtains a wave equation, which for the sideband modes can be Fourier analyzed and written as

$$[c^2 k_\pm^2 \mathbf{I} - \boldsymbol{\epsilon}_\pm \omega_\pm^2 - c^2 \boldsymbol{k}_\pm \boldsymbol{k}_\pm] \cdot \boldsymbol{E}_\pm(\omega_\pm, \boldsymbol{k}_\pm) = 4\pi i \omega_\pm \boldsymbol{j}_\pm(\omega_\pm, \boldsymbol{k}_\pm), \tag{8}$$

where $\mathbf{I}$ is the unit dyadic and $\boldsymbol{\epsilon}_\pm(\omega_\pm, \boldsymbol{k}_\pm)$ is the dielectric tensor of a magnetized electron-ion plasma [6].

The non-linear current density $\boldsymbol{j}_\pm$ produced by the interaction between the pump wave and the excited waves is

$$\boldsymbol{j}_\pm = q_\mathrm{e}(n_{\mathrm{e}l}^{(1)} \boldsymbol{v}_{0\pm}^{(0)} + n_{0\pm}^{(0)} \boldsymbol{v}_l^{(1)} + N_0 \boldsymbol{v}_\pm^{(1)}), \tag{9}$$

where N_0 is the unperturbed particle number density, $n_{0\pm}^{(0)}$ and $n_{\mathrm{e}l}^{(1)}$ are respectively the electron density perturbations associated with the electrostatic wave, $\boldsymbol{v}_{0\pm}^{(0)}$ and $\boldsymbol{v}_\pm^{(1)}$ are respectively the electron velocities in the pump and the scattered waves. The superscripts on $n_{\mathrm{e}l}$ and $\boldsymbol{v}_{0\pm,\pm,l}$ denote ordering with respect to the electric fields of the scattered waves. We note that the usual linear terms in the current density have already been included on the left-hand-side of Eq. (8), so that the latter yields the well-known linear dispersion relation [6] when $\boldsymbol{j}_\pm=0$. Ion contributions to the current density are negligible, as we are treating high-frequency waves. The second and third terms in Eq. (9) can be neglected [2] if the pump frequency is large $(\omega_0 \gg \omega_l, \omega_\mathrm{pe}$; ω_pe is the electron plasma frequency). The inhomogeneity is assumed to be weak, so that only the low frequency wave is affected.

Fourier analyzing Eqs. (2) and (6) and combining them, one obtains

$$n_{\mathrm{e}l}^{(1)} = (1+\chi_\mathrm{i})\, \boldsymbol{k}_l \cdot [N_0 \boldsymbol{v}_l^{(1)} + n_{0\pm}^{(0)} \boldsymbol{v}_\mp^{(1)} + n_\pm^{(1)} \boldsymbol{v}_{0\mp}^{(0)}]/\omega_l\, \hat{k}_l \cdot \boldsymbol{\epsilon}_l \cdot \hat{k}_l, \tag{10 a}$$

where

$$n_\pm^{(1)} = \boldsymbol{k}_\pm \cdot [N_0 \boldsymbol{v}_\pm^{(1)} + \boldsymbol{k}_l \cdot (N_0 \boldsymbol{v}_l^{(1)} + n_{0\mp}^{(0)} \boldsymbol{v}_\pm^{(1)})\, \boldsymbol{v}_{0\pm}^{(0)}/\omega_l + \boldsymbol{v}_{\mathrm{e}l}^{(1)} n_{0\pm}^{(0)}]/\omega_\pm [1 - \boldsymbol{k}_\pm \cdot \boldsymbol{v}_{0\pm}^{(0)}\, \boldsymbol{k}_l \cdot \boldsymbol{v}_{0\mp}^{(0)}/\omega_\pm \omega_l], \tag{10 b}$$

and χ_i is the ion susceptibility. The electron density perturbation $n_{0\pm}^{(0)}$ due to the pump field follows directly from the continuity equation, i.e., $n_{0\pm}^{(0)} = \boldsymbol{k}_{0\pm} \cdot \boldsymbol{v}_{0\pm}^{(0)}/\omega_{0\pm}$. We note that instead of using fluid equations as shown here, the low frequency mode can also be described by the ion and electron Vlasov equations [2], which cover a much wider range of validity. Thus, in the following, we shall occasionally use susceptibilities derived from the Vlasov system to describe the ES wave.

The velocities $\boldsymbol{v}_{0\pm}^{(0)}$, $\boldsymbol{v}_\pm^{(1)}$, and $\boldsymbol{v}_l^{(1)}$ can be obtained from Eqs. (2), (3), and (4). The result is [7]

$$\boldsymbol{v}_\mu^{(\alpha)} = -i(e/m_\mathrm{e}\omega_\mu)\{\boldsymbol{F}_\mu^{(\alpha)} + v_\mathrm{Te}^2 \boldsymbol{k}_\mu \times (\boldsymbol{k}_\mu \times \boldsymbol{F}_\mu^{(\alpha)})/\omega_\mu^2 - i\Omega_\mathrm{e} \boldsymbol{F}_\mu^{(\alpha)} \times (\hat{z} - k_{\mu z} v_\mathrm{Te}^2 \boldsymbol{k}_\mu/\omega_\mu^2)/\omega_\mu - F_{\mu z}^{(\alpha)} \hat{z}\, \Omega_\mathrm{e}^2/\omega_\mu\}/[1 - (\Omega_\mathrm{e}^2 + k_{\mu z}^2 v_\mathrm{Te}^2)/\omega_\mu^2 + k_{\mu z}^2 \Omega_\mathrm{e}^2 v_\mathrm{Te}^2/\omega_\mu^4]. \tag{11}$$

Here $\mu = 0\pm$, l, and $\pm$ stand for the pump, the ES, and the scattered wave respectively, and $\Omega_j = q_j B_0/m_j c$, $\omega_{0\pm} = \pm\omega_0$. We also define

$$\boldsymbol{F}_\mu^{(0)} = \boldsymbol{E}_\mu,$$

$$\boldsymbol{F}_\pm^{(1)} = \boldsymbol{v}_l^{(0)} \times (\boldsymbol{k}_{0\pm} \times \boldsymbol{E}_{0\pm})/\omega_{0\pm} + i m_\mathrm{e}[(\boldsymbol{k}_{0\pm} \cdot \boldsymbol{v}_l^{(0)})\boldsymbol{v}_{0\pm}^{(0)} + (\boldsymbol{k}_l \cdot \boldsymbol{v}_{0\pm}^{(0)})\boldsymbol{v}_l^{(0)}]/e,$$

$$\boldsymbol{F}_l^{(1)} = \boldsymbol{v}_{0+}^{(0)} \times (\boldsymbol{k}_- \times \boldsymbol{E}_-)/\omega_- + \boldsymbol{v}_{0-}^{(0)} \times (\boldsymbol{k}_+ \times \boldsymbol{E}_+)/\omega_+ + \boldsymbol{v}_+^{(0)} \times (\boldsymbol{k}_{0-} \times \boldsymbol{E}_{0-})/\omega_{0-} + \boldsymbol{v}_-^{(0)} \times (\boldsymbol{k}_{0+} \times \boldsymbol{E}_{0+})/\omega_{0+} + i(m_\mathrm{e}/e)[(\boldsymbol{k}_- \cdot \boldsymbol{v}_{0+}^{(0)})\boldsymbol{v}_-^{(0)} + (\boldsymbol{k}_+ \cdot \boldsymbol{v}_{0-}^{(0)})\boldsymbol{v}_+^{(0)} + (\boldsymbol{k}_{0+} \cdot \boldsymbol{v}_-^{(0)})\boldsymbol{v}_{0+}^{(0)} + (\boldsymbol{k}_{0-} \cdot \boldsymbol{v}_+^{(0)})\boldsymbol{v}_{0-}^{(0)}].$$

The non-linear forces $e\,\boldsymbol{F}^{(1)}_{l,\pm}/m_e$ originate from the $\boldsymbol{v}\times\boldsymbol{B}$ and $\boldsymbol{v}\cdot\nabla\boldsymbol{v}$ terms in the momentum Eq. (3), since the interaction of the pump with one of the scattered waves results in an oscillating field which is resonant with the other scattered wave.

The wave Eq. (8) for the scattered wave then becomes

$$
\begin{aligned}
&[c^2 k_\pm^2\,\mathbf{I} - \boldsymbol{\epsilon}_\pm\,\omega_\pm^2 - c^2 \boldsymbol{k}_\pm \boldsymbol{k}_\pm]\cdot\boldsymbol{E}_\pm = -\omega_{\mathrm{pe}}^2\,\omega_\pm\,(n_{el}^{(1)}/N_0)\,\{[\boldsymbol{F}_{0\pm}^{(0)} + v_{\mathrm{Te}}^2\,\boldsymbol{k}_{0\pm}\times(\boldsymbol{k}_{0\pm}\times\boldsymbol{F}_{0\pm}^{(0)})/\omega_{0\pm}^2 \\
&\quad - i\,\Omega_e\,\boldsymbol{F}_{0\pm}^{(0)}\times(\hat{z} - k_{0\pm z}\,v_{\mathrm{Te}}^2\,\boldsymbol{k}_{0\pm}/\omega_{0\pm}^2)/\omega_{0\pm} - F_{0\pm z}^{(0)}\,z\,\Omega_e^2/\omega_{0\pm}]/\omega_{0\pm}\,[1 - (\Omega_e^2 + k_{0\pm z}^2\,v_{\mathrm{Te}}^2)/\omega_{0\pm}^2 \\
&\quad + k_{0\pm z}^2\,\Omega_e^2\,v_{\mathrm{Te}}^2/\omega_{0\pm}^4]\} - \omega_{\mathrm{pe}}^2\,\omega_\pm\,(n_{0\pm}^{(0)}/N_0)\,\{0\pm\to l;\ \boldsymbol{F}_{0\pm}^{(0)}\to\boldsymbol{F}_l^{(1)}\} - \omega_{\mathrm{pe}}^2\,\omega_\pm\,\{0\pm\to\pm;\ \boldsymbol{F}_{0\pm}^{(0)}\to\boldsymbol{F}_\pm^{(1)}\},
\end{aligned}
\tag{12}
$$

where the last two brackets are identical to the first one except for the indicated changes. The above equation, together with Eq. (10), can be used to investigate the decay of a high frequency EM-ES wave propagating at any angle to the external magnetic field into a scattered EM-ES wave and an ES wave.

In the following, we shall discuss scattering and modulational instabilities for linearly polarized EM waves propagating perpendicular to the external magnetic field. The pump and the scattered wave shall be assumed to be ordinary waves ($\boldsymbol{E}\parallel\boldsymbol{B}_0$). For scattering instabilities the pump and the ES waves are approximate solutions of the appropriate linear dispersion relations, while for modulational instabilities both sidebands satisfy approximately the linear dispersion relation and they lead to long-wavelength modulations of the particle density. In the latter case, the frequency and the wavelength of the density modulations do not generally satisfy the linear dispersion relation.

The perturbed electron density $n_{el}^{(1)}$ for waves propagating transverse to the magnetic field can be obtained from Eqs. (10) and (11), i.e.,

$$
n_{el}^{(1)} = i\,e\,N_0\,\boldsymbol{k}_l\cdot\boldsymbol{F}_l^{(1)}\,\chi_e(1+\chi_i)/m_e\,\omega_{\mathrm{pe}}^2\,(1+\chi_e+\chi_i)\,,
\tag{13}
$$

where χ_e is the electron susceptibility.

From Eq. (11) one obtains the induced electron velocity in the pump field

$$
\boldsymbol{v}_{0\pm}^{(0)} = -i\,e\,\boldsymbol{E}_{0\pm}/m_e\,\omega_{0\pm}\,,
\tag{14}
$$

where the magnetic field of the pump has been neglected, since we are treating non-relativistic effects only. The ponderomotive force originating from the beating of the pump and the sideband EM waves is given by

$$
\begin{aligned}
(e/m_e)\,\boldsymbol{F}_l^{(1)} = -i\,e\,\boldsymbol{k}_l[&E_{0+}\,E_-/\omega_- \\
&- E_{0-}\,E_+/\omega_+]/m_e\,\omega_0\,.
\end{aligned}
\tag{15}
$$

Combining Eqs. (8), (9), (13), and (15) we get for $\omega_0 \gg \omega_{\mathrm{pe}}$,

$$
\begin{aligned}
E_\pm = \mp e^2\,k_l^2\,\omega_\pm\,\chi_e(1+\chi_i)E_{0\pm}\,(&E_{0+}\,E_-/\omega_- \\
&- E_{0-}\,E_+/\omega_+)/m_e^2\,\omega_0^2\,(1+\chi_e+\chi_i)D_\pm\,,
\end{aligned}
\tag{16}
$$

where

$$
D_\pm = c^2 k_\pm^2 - \omega_\pm^2 + \omega_{\mathrm{pe}}^2\,.
$$

Equation (16) represents a set of coupled equations for E_+ and E_-, from which one readily obtains the dispersion relation

$$
\chi_e^{-1} + (1+\chi_i)^{-1} = V_0^2\,k_l^2\,(D_-^{-1} + D_+^{-1})\,,
\tag{17}
$$

where $V_0 = e\,E_0/m_e\,\omega_0$.

We note that Eq. (17) is also valid for the situation in which the waves are in arbitrary directions with respect to $\boldsymbol{B}_0$, but the pump frequency is much larger than the electron gyrofrequency [8]. This is due to the fact that in the latter, as well as in our case, the high-frequency waves are not influenced by the external magnetic field.

3. The Scattering and Modulational Instabilities

We first consider stimulated scattering processes in which the Stokes component is resonant ($D_- \approx 0$), while the anti-Stokes component is not ($D_+ \neq 0$). This approximation is valid for not too small k_l values. Furthermore, the condition $D_- \approx 0$ can be satisfied for $k_l = 2\,\mathrm{k}_0\cos\Theta$, where Θ is the angle between $\boldsymbol{k}_l$ and $\boldsymbol{k}_0$. Under these assumptions the term containing D_+ in Eq. (17) can be neglected. In the following, several cases of special interest are considered.

First, we investigate scattering off waves near the upper hybrid frequency. These waves propagate almost perpendicular to the external magnetic field. They can be approximated as ES waves [6] as long as the frequency ω_l is close to the upper hybrid resonance frequency $\omega_{\mathrm{U}} = (\omega_{\mathrm{pe}}^2 + \Omega_e^2)^{1/2}$. The ion response can be neglected because of the high-frequency. For the same reason, the electron inertia term dominates over the pressure term. Letting

$\omega_l = \omega_{\rm U} + i(\gamma_{\rm U} + \Gamma_{\rm U})$, and assuming $\chi_j = -\omega_{\rm pj}^2/(\omega^2 - \Omega_j^2)$, we obtain for $\omega \gg \Omega_{\rm i}$, $\omega_{\rm pi}$ and $\omega_0 \approx c\,k_0 > \omega_{\rm U}$ the growth rate

$$\gamma_{\rm U} = -(\Gamma_{\rm U} + \Gamma_-)/2 \pm [(\Gamma_{\rm U} - \Gamma_-)^2/4 + \cos^2\Theta\, V_0^2\,\omega_0\,\omega_{\rm pe}^2/c^2(\omega_{\rm pe}^2 + \Omega_{\rm e}^2)^{1/2}]^{1/2}, \qquad (18)$$

where the frequency shift due to the pump wave has been neglected. We have introduced the damping rates $\Gamma_{\rm U}$ and Γ_- of the low frequency and the scattered EM waves. The maximum growth rate occurs for backscattering, i. e., $\Theta = 0$ and $k_l \approx 2\,k_0$. The threshold is

$$E_{0\rm U}^2 = m_{\rm e}^2\,\omega_0\,\Gamma_{\rm U}\,\Gamma_-\,c^2(\omega_{\rm pe}^2 + \Omega_{\rm e}^2)^{1/2}/e^2\,\omega_{\rm pe}^2. \qquad (19)$$

This result differs from that of Ref. 8, in which an algebraic error seems to have occurred.

Next, we consider scattering off waves near the lower hybrid frequency. These waves also propagate almost perpendicular to the external magnetic field. They can be approximated as ES waves if the frequency is close to the lower hybrid resonance frequency $\omega_{\rm L} = |\Omega_{\rm i}\,\Omega_{\rm e}|^{1/2}$, since then the transverse component of the field is of order $(m_{\rm e}/m_{\rm i})^{1/2}$ less than the longitudinal component[6]. As low frequency waves are involved here, the ion dynamics can no longer be completely neglected. For $\Omega_{\rm i} < \omega_l < \Omega_{\rm e} < \omega_{\rm pe,i}$, we have $\chi_{\rm e} = \omega_{\rm pe}^2/\Omega_{\rm e}^2$ and $\chi_{\rm i} = -\omega_{\rm pi}^2/\omega^2$. The effect of the ponderomotive force $e\,\mathbf{F}_l^{(1)}/m_{\rm i}$ on the ions can be neglected as it is smaller by a mass ratio to that of the electrons.

By means of a procedure similar to that used above, letting $\omega_l \approx \omega_{\rm L} + i(\gamma_{\rm L} + \Gamma_{\rm L})$, where $\Gamma_{\rm L}$ is the damping rate of the wave near the lower hybrid frequency, one obtains the growth rate

$$\gamma_{\rm L} = -(\Gamma_{\rm L} + \Gamma_-)/2 \pm [(\Gamma_{\rm L} - \Gamma_-)^2/4 + \cos^2\Theta\, V_0^2\,\omega_{\rm pi}^2\,\omega_0/c^2\,|\Omega_{\rm i}\,\Omega_{\rm e}|^{1/2}]^{1/2}. \qquad (20)$$

The growth rate is again maximum for backscattering. The threshold is given by

$$E_{0\rm L}^2 = m_{\rm e}^2\,\omega_0\,\Gamma_{\rm L}\,\Gamma_-\,|\Omega_{\rm e}\,\Omega_{\rm i}|^{1/2}\,c^2/\omega_{\rm pi}^2\,e^2. \qquad (21)$$

The above result agrees with that of Reference 8.

If the plasma contains a weak inhomogeneity, the secondary EM wave can also scatter off low frequency drift waves. Assuming that the density gradient is $L^{-1} = \hat{e}_x\,{\rm d}(\ln N_0)/{\rm d}x$, we consider two separate cases: first, drift wave propagation perpendicular to $\mathbf{B}_0$ in a low β plasma, and second, drift wave propagating *almost* but not exactly perpendicular to $\mathbf{B}_0$. In both cases we use the local approximation and include first-order effects of the diamagnetic drift[6, 9].

For low frequency drift waves propagating perpendicular to $\mathbf{B}_0$ in a low β plasma ($\omega \ll \Omega_{\rm i}$, $k_{lz} = 0$, $\mathbf{k}_l \approx k_l\,\hat{e}_y$), the susceptibilities can then be approximated in the long-wave-length limit $k_l\,v_{{\rm T}j} > \Omega_j$,

$$\chi_j = [1 - (1 + \omega_j^*/\omega)\,(1 - k_l^2\,v_{{\rm T}j}^2/\Omega_j^2)]/k_l^2\,\lambda_j^2,$$

where λ_j is the Debye length and $\omega_j^* = k_{ly}\,v_{Tj}^2/L\,\Omega_j$ is the diamagnetic drift frequency of component j. In this case the ES mode is a drift wave moving approximately in the direction of the drift with nearly the speed of the ion fluid velocity. Its frequency is

$$\omega_{\rm D} = \sum_j \omega_j^*\,(k_l^{-2}\,\lambda_j^{-2} - \omega_{{\rm p}j}^2/\Omega_j^2)/(1 + \sum_j \omega_{{\rm p}j}^2/\Omega_j^2).$$

The wave is weakly Landau damped but can be driven linearly unstable by a gravitational force opposite to the density gradient or by temperature gradients. Assuming $\omega_l = \omega_{\rm D} + i(\gamma_{\rm D} + \Gamma_{\rm D})$ and following the procedure outlined above, one can investigate the corresponding instability. It turns out that the threshold

$$E_{0\rm D}^2 = 2\,\omega_0^3\,m_{\rm e}^2 \sum_j \omega_j^*\,(\omega_{{\rm p}j}^2/\Omega_j^2 - k_l^{-2}\,\lambda_j^{-2})\,\Gamma_-\,\Gamma_{\rm D}/e^2\,k_l^2\,\omega_{\rm D}^2\,[\chi_{\rm e}(1 + \chi_{\rm i})]_{\omega = \omega_{\rm D}} \qquad (22)$$

is rather large for small electron to ion mass ratio, so that this instability is of little physical significance.

For low frequency drift waves propagating nearly perpendicular to $\mathbf{B}_0$, we use $\chi_{\rm e} = 1/k_l^2\,\lambda_{\rm e}^2$, and $\chi_{\rm i} = \omega_{\rm i}^*/\omega_l\,k_l^2\,\lambda_{\rm i}^2$. We have assumed that a small component k_{lz} parallel to $\mathbf{B}_0$ of the ES wave vector $\mathbf{k}_l$ exists, so that $k_{lz}\,v_{\rm Ti} < \omega < k_{lz}\,v_{\rm Te}$. The momentum conservation relation $k_{0z} \approx 0 \approx k_{lz} - k_{-z}$ then requires k_{-z} to be finite. However, if $k_{-z} = k_{lz}$ is sufficiently small, the scattered wave can still be approximated as purely electromagnetic. In this case the ES mode is the drift wave, propagating with a frequency $\omega_{\rm d} = \omega_{\rm e}^*/(1 + k_l^2\,\lambda_{\rm e}^2)$. This wave is linearly unstable (due to Landau growth, which is not considered in our macroscopic derivation, but must nevertheless be phenomenologically included) if $k_{lz} \neq 0$. Here, we assume that k_{lz} is so small that collisional effects cause a net damping $\Gamma_{\rm d}$ of the drift wave. Assuming $\omega_l = \omega_{\rm d} + i(\gamma_{\rm d} + \Gamma_{\rm d})$, and following the procedure used above, we obtain

$$\gamma_{\rm d} = -(\Gamma_{\rm d}+\Gamma_{-})/2 \pm [(\Gamma_{\rm d}-\Gamma_{-})^2/4 + k_{ly} v_{\rm Te}^2 V_0^2/2 L \Omega_{\rm e} \omega_0 \lambda_{\rm e}^2 (1+k_l^2\lambda_{\rm e}^2)^2]^{1/2}. \tag{23}$$

The corresponding threshold is given by

$$E_{0\rm d}^2 = 2\Gamma_{\rm d}\Gamma_{-} m_{\rm e}^2 \omega_0^3 \Omega_{\rm e} \lambda_{\rm e}^2 (1+k_l^2\lambda_{\rm e}^2)^2 L/e^2 k_{ly} v_{\rm Te}^2. \tag{24}$$

Since drift waves propagate predominantly perpendicular to both the density gradient and the magnetic field, the direction of the scattered wave depends on the direction of the pump wave. Thus, if the pump is colinear with the density gradient, there will be side-scattering with a maximum for $\Theta = \pi/4$ if $k_l \lambda_{\rm e} \ll 1$.

In contrast to the scattering instabilities, where only the Stokes component is considered, for modulational instabilities both Stokes and anti-Stokes components are included on the same footing. The corresponding density modulations, which in general do not satisfy any linear dispersion relation, are of low frequency and long wavelength. Modulational instabilities are important as they can lead to the phenomena of light trapping and filamentation within the plasma and cause localized heating [9].

From Eq. (17) one obtains

$$\chi_{\rm e}^{-1} + (1+\chi_{\rm i})^{-1} = -2V_0^2\delta^2/c^2[(\omega_l - \boldsymbol{k}_l\cdot\boldsymbol{v}_{\rm g})^2 - \delta^2], \tag{25}$$

where $\delta = c^2 k_l^2/2\omega_0$, and $\boldsymbol{v}_{\rm g} = c^2\boldsymbol{k}_0/\omega_0$ is the group velocity of the pump wave.

Similar to the case of scattering off drift waves, we introduce a finite but small parallel (to $\boldsymbol{B}_0$) component of the ES wave vector, satisfying $k_{lz} \ll k_{l\perp}$. Then, for $\omega \ll k_{lz} v_{\rm Ti}$ we have $\chi_j = 1/k_l^2\lambda_j^2$. After substituting into Eq. (25), we find that for $\boldsymbol{k}_l\cdot\boldsymbol{v}_{\rm g} = 0$ a purely growing instability can occur. The minimum threshold is the same as in Reference 2. This is expected because here none of the modes is affected by the presence of the magnetic field.

A nontrivial case exists if $k_{lz} v_{\rm Ti} \ll \omega \ll k_{lz} v_{\rm Te}$, whence $\chi_{\rm e} = k_l^{-2}\lambda_{\rm e}^{-2}$ and $\chi_{\rm i} = -\omega_{\rm pi}^2/(\omega^2 - \Omega_{\rm i}^2)$. Inserting these values for the susceptibilities into Eq. (25) we get, for $\boldsymbol{k}_l\cdot\boldsymbol{v}_{\rm g} = 0$, straightforwardly

$$\omega_l^2 = (A+\delta^2)/2 \pm [(A+\delta^2)^2/4 + 2V_0^2\delta^2(\omega_{\rm pi}^2 + \Omega_{\rm i}^2)/c^2(1+k_l^2\lambda_{\rm e}^2)]^{1/2}, \tag{26}$$

which determines the onset and growth rate of the modulational instability. Here, $A = [(\omega_{\rm pi}^2 + \Omega_{\rm i}^2)k_l^2\lambda_{\rm e}^2 + \Omega_{\rm i}^2]/[1+k_l^2\lambda_{\rm e}^2]$, and the high-frequency waves are assumed to be only slightly damped.

For $k_l^2\lambda_{\rm e}^2 \ll 1$ and $\delta^2 \gg A$, we obtain a typical growth rate $\gamma \approx V_0(\omega_{\rm pi}^2 + \Omega_{\rm i}^2)^{1/2}/c$. Thus the magnetic field becomes important for $\Omega_{\rm i} \geqq \omega_{\rm pi}$. Other modulational instabilities corresponding to different frequency and wavelength regions can readily be investigated within the above formulation.

4. Discussion

In this paper, we have considered the decay of a high-frequency EM or mixed EM-ES wave into a scattered EM or EM-ES wave and an ES wave in magnetized, nearly homogeneous plasmas. An equation is derived for the scattered wave in terms of the non-linear forces originating from the interaction between the pump wave and the ES wave. The formulation is valid for the propagation of waves in arbitrary directions. For the case in which all the wave vectors are almost perpendicular to the external magnetic field [5], we have calculated the growth rates and the thresholds for the stimulated scattering and modulational instabilities.

The effect of the magnetic field on the scattering instabilities in homogeneous plasmas is to enhance the threshold. This can easily be shown by comparing our results with those of unmagnetized plasmas [2]. The comparison yields the ratio $E_{0\rm U}^2/E_{0\rm R}^2 \approx (1+\Omega_{\rm e}^2/\omega_{\rm pe}^2)^{1/2}$ between the threshold for scattering off upper hybrid modes in a magnetized plasma and the corresponding threshold $E_{0\rm R}^2$ for scattering off plasma waves in unmagnetized plasmas. Since the threshold for scattering off waves near the upper hybrid frequency is smaller by a factor $[(1+\omega_{\rm pe}^2/\Omega_{\rm e}^2)m_{\rm e}/m_{\rm i}]^{1/2}$ compared to that of scattering off the lower-hybrid mode, we have accordingly $E_{0\rm L}^2/E_{0\rm R}^2 \approx \Omega_{\rm e}/\omega_{\rm pi}$.

New features appear in magnetized plasmas with a small density gradient, as drift waves can be excited. The threshold ratio E_{0d}^2/E_{0R}^2 can be estimated to be $k_0 L \Omega_e/\omega_{pe}$, which depends on both the strength of the inhomogeneity (L^{-1}) and the external magnetic field. Due to the local approximation used in the drift wave calculations, $k_0 L$ must be large, so that only for small Ω_e/ω_{pe} scattering off drift waves is significant.

As in the case of an unmagnetized plasma, the maximum growth rate of the scattering instabilities in a homogenoeus magnetized plasma occurs for backscattering. One concludes that in magnetized plasmas, as well as in unmagnetized ones, stimulated backscattering can be a competing process to the other parametric instabilities in which an EM wave decays into two ES waves [5, 10–12].

We have also investigated modulational instabilities in which the plasma density is modulated on a long spatial scalelength. For the case of interest, the ratio of the typical growth rates with and without the external magnetic field is $(1+\Omega_i^2/\omega_{pi}^2)^{1/2}$. Hence the latter can enhance the growth rate.

External magnetic field effects may be important in studying laser irradiated plasmas, in which a D.C. magnetic field [13, 14] can be excited in the corona due to thermoelectric effects or the $\nabla n \times \nabla T$ currents arising from asymmetries. It is also important in investigating heating experiments in the ionosphere, where the geomagnetic field is ever present.

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